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Wave effects on coastal and offshore structures have been studied for more than two decades. Nevertheless, the dynamic behaviors of those structures due to a real sea state, considering the features of randomness, nonlinearity, and directionality, have seldom been investigated. In this report, random and nonlinear wave effects on those structures have been examined analytically.

The report consists of three parts. The first part presents a new model of a linear random wave force transformation on a sloping beach as the waves propagate from offshore toward the shore. The second part investigates the effect of nonlinear wave interactions on the dynamic response of a fixed offshore structure. The third part examines the effect of wave groups and an associated forced second-order wave system on the dynamic response of a fixed offshore structure. The results of this report show that random and nonlinear waves play important roles on the dynamic behaviors of coastal and offshore structures.

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The Effects of Random and Nonlinear Waves on Coastal and Offshore Structures

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Foreword

The mission of the Naval Ocean Research and Development Activity is to carry out a broadly based RDT&E program in ocean science and technology. To this end, NORDA is investigating the effects of the ocean environment on a variety of naval structures. In particular, this report shows the important role played by random and nonlinear waves on the dynamic behavior of coastal and offshore structures.

A. C. Esau, Captain, USN
Commanding Officer, NORDA

Executive summary

Wave effects on coastal and offshore structures have been studied for more than two decades. Nevertheless, the dynamic behaviors of those structures due to a real sea state, considering the features of randomness, nonlinearity, and directionality, have seldom been investigated. In this report, random and nonlinear wave effects on those structures have been examined analytically.

The appendix contains three articles. The first presents a new model of a linear random wave force transformation on a sloping beach as the waves propagate from offshore toward the shore. The second investigates the effect of nonlinear wave interactions on the dynamic response of a fixed offshore structure. The third examines the effect of nonlinear wave groups and an associated forced second-order wave system on the dynamic response of a fixed offshore structure. The results of these documents show that random and nonlinear waves play important roles on the dynamic behaviors of coastal and offshore structures.

Acknowledgments

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Contents

The effects of random and nonlinear waves on coastal and offshore structures

Introduction

Over the past two decades, ocean engineers and oceanographers have confronted a long-term survival using such coastal and offshore structures as breakwaters, moored ships, and sensor platforms in extreme seas. Nevertheless, most of these structures may sustain severe damage due to hurricanes or storms sweeping through a coastal region. To design these structures for satisfactory operation during severe wave conditions, better definitions of environmental forcing functions, i.e., a real sea state considering randomness, nonlinearity, and directionality, are needed. In the report appendices random and nonlinear wave effects on these structures are examined analytically in three articles.

The first article (presented at *ASCE Engineering Mechanics Division Specialty Conference*, Laramie, Wyoming, August 1984), based on the random process approach, presents the development of a linear random wave force transformation model, which predicts the random wave force in shallow water from a given storm-generated wave field in deep water. For a coastal region, this model, taking into account the sloping beach condition, is more realistic than the current model with a locally uniform flat bottom

The second article (presented at *Oceans '84* Conference, Washington, D.C., September 1984), based on the Morison's equation approach investigates the dynamic response of a single degree of freedom offshore structure using nonlinear wave interactions of two deep-water waves. This model, which calculates wave forces on offshore structures, is more accurate than most of the current models treating waves as a superposition of linear wave components.

The third article (presented at *OCEANS '86* Conference, Washington, D.C., September 1986) is based on the Morison's equation approach and examines the effect of wave groups and an associated force second-order wave system on a single degree of freedom offshore structure. It is mainly because large waves in the groups can cause more structural damage than individual waves of the same size and dispersed throughout a wave train. Therefore, the wave group effects have to be considered for design purpose.

Conclusions and recommendations

The effects of random and nonlinear waves on the dynamic behaviors of coastal and offshore structures have been examined analytically. These waves have been shown to play important roles on generating structural response. Therefore, the results of this report can provide ocean engineers with a superior and standard basis for calculating dynamic response of coastal and offshore structures. It is also expected that the results can provide the Navy with an overall and long-term safety and cost reduction in coastal and offshore structure design. Several conclusion from this report follow.

- A linear random wave force transformation model on a sloping beach is developed. This model improves the current approximate model because the assumption of a locally flat bottom is removed. Though the results show only in 45^0 sloping beach, they can extend to $\pi/2n$ sloping beach (with n an integer). For engineering application, this model is used to predict the wave force spectrum at any specified location in the coastal region from a given deep-water wave energy spectrum.

- The results of the effect of the nonlinear wave interactions of deep water show that the second-order waves (cross interactions) make only a small modification to the structural response. The third-order waves (resonant

interactions), however, produce a significant effect because they grow in time. The structural response due to the effect of the third-order waves at the wave crest phase decreases with increasing time of resonant interactions. At mean water level phase, the structural response with the third-order wave effects increases as the time scale increases. For both wave phases, the variations of the structural response as $\sigma_2<\sigma_1$ is larger than those as $\sigma_2>\sigma_1$. Also, the variations vanish when the two wave trains are parallel or antiparallel.

• The results of the effect of second-order wave system on offshore structures are particularly important in shallow water. Structural response due to drag force and inertia force from the combined effects of the primary and the second-order wave systems are found to be reduced at the wave crest position and the mean water level. The reduction of structural response due to inertia force is much smaller than that due to drag force because the drag force includes a square term. The structural response due to drag force at trough position has not been shown in the results. Clearly, the results would be an increase of structural response at the trough phase position. However, it is because the magnitude of the structural response at crest position is greater than that at trough position due to the additional submergence of piling at the crest position. Therefore, only the crest position was considered.

Several recommendations are made for further study of this topic. The directional wave effect needs to be considered. Since the wave energy distributes unevenly in different directions, it can cause structural damage in specific direction.

• A numerical model of a real sea state on the dynamic behavior of coastal and offshore structures needs to be developed. This model can simulate a real sea state for practical use; however, the analytical model provides only certain features because of the limitations of the analytical tools.

• Model and field experiments need to be performed, mainly for verifying the theoretical results.

Appendix A: The effect of nonlinear wave interactions on an offshore structure

THE EFFECT OF NONLINEAR WAVE INTERACTIONS ON AN OFFSHORE STRUCTURE

Benjamin Yun-Hai Chen and Darrell A. Milburn

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ABSTRACT

The effect of nonlinear wave interactions on a fixed offshore structure is examined analytically. Based on Morison's equation, the dynamic response of a single degree of freedom structure is investigated using the linear (first-order waves) and nonlinear wave interactions (second- and third-order waves) generated by two deepwater wave trains. By comparing the results, it is found that the third-order waves (resonant interactions) can significantly affect structural response.

1. INTRODUCTION

Most models for calculating wave force effects on offshore structures treat the waves as a superposition of linear wave components. Examples include the works of Arockiasamy, et al. [1], Barik and Paramasivam [2], Dao and Penzien [3], Leonard, et al. [4], and Tuah and Hudspeth [8]. For a real sea state, the superposition of linear wave components cannot satisfy the required degree of accuracy; hence, the features of nonlinear wave interactions have to be considered.

In small amplitude, first-order wave theory, two or more simple wave trains are propagated independently and without mutual interaction. In the second-order wave theory, the interactions, including self-interactions and cross-interactions, produce only a small modification to the wave motion which remains bounded in time. Phillips [6] found that three primary waves can transfer energy to a fourth wave in such a way that the amplitude of the fourth wave increases linearly with time. Since these interactions occur in the oceans, they may produce a considerable modification in the ocean wave spectra. One particular case of Phillip's results was found by Longuet-Higgins [5] when two of the three primary waves have the same wave number.

The effect of these nonlinear wave interactions on the dynamic response of a fixed offshore structure is examined analytically in this paper. For simplicity, a single degree of freedom structure with negligible wave scattering is considered. Also, the wave loads are determined by applying Morison's equation which includes both drag and inertial force components.

In the next section, the theory of nonlinear wave interactions will be described. The structural response with and without the presence of the nonlinear wave interaction effects will be derived in section 3. The results will be shown in section 4.

2. THEORY OF NONLINEAR WAVE INTERACTIONS

The velocity potential of the nonlinear wave interaction theory has been developed by Longuet-Higgins. For the wave force consideration, the authors have extended this theory to wave kinematics, *i.e.*, wave velocity and acceleration. The theory is given below.

First-Order Waves

Since the two wave trains considered are in deep water, the velocity potential of the two wave trains can be given as

$$\phi_i = a_i\, \sigma_i\, k_i^{-1}\, e^{k_i Z} \sin \psi_i \tag{1}$$

Where $k_i (= |\vec{k_i}|)$ is the wave number, σ_i the angular frequency, a_i the wave amplitude. The phase function, ψ_i, is defined as $\psi_i = \vec{k_i} \cdot \vec{x} - \sigma_i t$ where t is time and $\vec{x}$ is a vector representation of the two horizontal directions x and y. The origin of the coordinate system is located at the mean water surface with Z positive upward. "i = 1" represents the first wave train and "i = 2", the second wave train.

The horizontal velocity, u_i, and acceleration, $\dot{u}_i$, of the wave trains can be computed as

$$u_i = \nabla_H\, \phi_i = a_i\, \sigma_i\, e^{k_i Z} \cos \psi_i \tag{2}$$

and

$$\dot{u}_i = \frac{\partial u_i}{\partial t} = a_i\, \sigma_i^2\, e^{k_i Z} \sin \psi_i \tag{3}$$

where ∇_H is the spatial graident in the horizontal direction.

Second-Order Waves

The second-order waves of the two deepwater wave trains are comprised of self-interaction and cross-interaction components. For a single irrotational wave in deep water, the velocity and acceleration of the self-interaction components vanish. Hence

$$u_{20} = \nabla_H\, \phi_{20} = 0 \tag{4}$$

and

$$u_{02} = \nabla_H\, \phi_{02} = 0\ . \tag{5}$$

and only the cross-interaction components remain. The velocity potential of the cross-interaction components, ϕ_{11}, is given as

$$\phi_{11} = A\, e^{|\vec{k_1} - \vec{k_2}|Z} \sin(\psi_1 - \psi_2) - B\, e^{|\vec{k_1} + \vec{k_2}|Z} \sin(\psi_1 + \psi_2) \tag{6}$$

where

$$A = \frac{2\, a_1\, a_2\, \sigma_1\, \sigma_2\, (\sigma_1 - \sigma_2) \cos^2 \tfrac{1}{2}\theta}{(\sigma_1 - \sigma_2)^2 - g\, |\vec{k_1} - \vec{k_2}|} \tag{7}$$

$$B = \frac{2\, a_1\, a_2\, \sigma_1\, \sigma_2\, (\sigma_1 + \sigma_2) \sin^2 \tfrac{1}{2}\theta}{(\sigma_1 + \sigma_2)^2 - g\, |\vec{k_1} + \vec{k_2}|} \tag{8}$$

Here θ is the angle between the two wave trains (see Figure 1) and g is the acceleration of gravity.

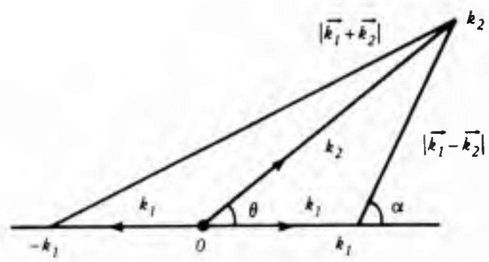

Figure 1. Definition for θ.

The horizontal velocity, u_{11}, and acceleration, $\dot{u}_{11}$, are derived from Eq. (6) as

$$u_{11} = |\vec{k_1} - \vec{k_2}|\, A\, e^{|\vec{k_1} - \vec{k_2}| Z} \cos(\psi_1 - \psi_2)$$

$$- |\vec{k_1} + \vec{k_2}|\, B\, e^{|\vec{k_1} + \vec{k_2}| Z} \cos(\psi_1 + \psi_2) \qquad (9)$$

and

$$\dot{u}_{11} = (\sigma_1 - \sigma_2)\, |\vec{k_1} - \vec{k_2}|\, A\, e^{|\vec{k_1} - \vec{k_2}| Z} \sin(\psi_1 - \psi_2)$$

$$- (\sigma_1 + \sigma_2)\, |\vec{k_1} + \vec{k_2}|\, B\, e^{|\vec{k_1} + \vec{k_2}| Z} \sin(\psi_1 + \psi_2) \qquad (10)$$

Third-Order Waves

The third-order waves of the two deepwater wave trains generate the resonant interaction components which grow in time. The velocity potential, ϕ_{21}, of the resonant interaction components is

$$\phi_{21} = - \frac{Pt}{2\,(2\sigma_1 - \sigma_2)}\, e^{|2\vec{k_1} - \vec{k_2}| Z} \cos(2\psi_1 - \psi_2) \qquad (11)$$

where Longuet-Higgins defines P as

$$P = (a_1 k_1)^2 (a_2 k_2)\, g^2\, \sigma_1^{-1}\, F(\epsilon) \qquad (12)$$

and

$$F(\epsilon) = \frac{(1 + \frac{1}{2}\epsilon^2)^2 (1 - 4\epsilon^2)}{(1 + \epsilon)^3} \left[1 + \frac{4\epsilon}{\epsilon - (6 + \epsilon^2)^{\frac{1}{2}}} \right] \qquad (13)$$

$$\epsilon = \frac{\sigma_2 - \sigma_1}{\sigma_1} \qquad (14)$$

$F(\epsilon)$ is called the coupling factor. Eq. (12) is a compact form of Eq. (4.2) in Longuet-Higgin's paper.

Based on the work of Longuet-Higgin's [5], the resonant interactions occur when the two deepwater wave trains give a locus of a "figure-of-eight" (see Figure 2). Positive values of ϵ correspond to points on the right-hand loop, and negative values of ϵ to points on the left-hand loop.

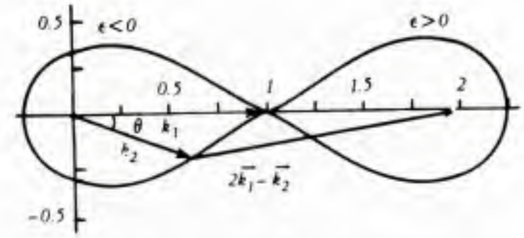

Figure 2. The resonance loop for third-order binary interactions.

The horizontal velocity, u_{21}, and acceleration, $\dot{u}_{21}$, derived from Eq. (11) are

$$u_{21} = \frac{P\,|2\vec{k_1} - \vec{k_2}|\, t}{2\,(2\sigma_1 - \sigma_2)}\, e^{|2\vec{k_1} - \vec{k_2}| Z} \sin(2\psi_1 - \psi_2) \qquad (15)$$

and

$$\dot{u}_{21} = - \frac{P\,|2\vec{k_1} - \vec{k_2}|\, t}{2}\, e^{|2\vec{k_1} - \vec{k_2}| Z} \cos(2\psi_1 - \psi_2)$$

$$+ \frac{P\,|2\vec{k_1} - \vec{k_2}|}{2\,(2\sigma_1 - \sigma_2)}\, e^{|2\vec{k_1} - \vec{k_2}| Z} \sin(2\psi_1 - \psi_2) \qquad (16)$$

By comparing Eqs. (15), (16), (2), (3), (9), and (10), it can be seen that the third-order waves are out of phase with those of first- and second-order waves. The first term of Eq. (16) is a function of time and the other is not. Because the first term dominates with increasing time, the other term will be neglected.

Based on the theory described above, we can now analyze the structural response due to these nonlinear wave interaction effects.

3. NONLINEAR WAVE INTERACTION EFFECTS ON A SIMPLE STRUCTURE

By choosing a generalized single degree of freedom structure, one can easily study its response to the nonlinear wave interaction effects derived. For such a structure, the equation of motion is

$$\ddot{\zeta}(t) + 2\omega_0 \lambda \dot{\zeta}(t) + \omega_0^2 \zeta(t) = \frac{1}{M} F_W(t) \qquad (17)$$

where $\lambda = C/2M\omega_0$ and $\omega_0 = (K/M)^{\frac{1}{2}}$. Here M is the mass of structure, C is the internal structural damping, K is the structural stiffness, ζ, $\dot{\zeta}$, and $\ddot{\zeta}$ are the displacement, velocity and acceleration of the structure along the resultant wave direction, and $F_W(t)$ is the external force acting on the structure. All the above quantities are based on a unit length of the structure.

For most wave loading conditions, the water particle motion is much larger than the structural motion. Therefore, it is reasonable to assume that the only external force acting on the structure is due to wave motions and that $F_W(t)$ can be represented by Morison's equation

$$F_W(t) = \frac{\varrho}{2} C_D\, D\, u(t)\, |u(t)| + \varrho\, C_M \frac{\pi D^2}{4} \dot{u}(t) \qquad (18)$$

where C_D is the drag coefficient, C_M is the inertia coefficient, ϱ is the mass density of water, D is the pile diameter, and u and $\dot{u}$ are the local water particle velocity and acceleration.

The dynamic response of a fixed offshore structure with and without nonlinear wave interaction effects (*i.e.*, second-order cross interactions and third-order resonant interactions) will be derived as follows:

First-Order Motions

The water particle velocity, u, of the two first-order waves is $(\dot{u}_1 + u_2)$, and the water particle acceleration, $\dot{u}$, is $(\dot{u}_1 + \dot{u}_2)$. If the wave crest phase of the two wave trains is selected, there is only drag force (*i.e.*, the first term on the right-hand side of Eq. (18)). For the mean water level (*i.e.*, z = 0), the structural response due to the drag force is derived by inserting Eq. (18) into Eq. (17). The result is

$$\zeta_{1c} = \frac{\varrho}{2\,M\omega_O^2} C_D\, D\,(a_1\,\sigma_1 + a_2\,\sigma_2)^2 \tag{19}$$

If the mean water level phase of the two wave trains is selected, inertia force (*i.e.*, the second term on the right-hand side of Eq. (18)) survives. Similarly, for the position at mean water level (z = 0), the structural response due to the inertial force is

$$\zeta_{1m} = \frac{\varrho}{M\omega_O^2} C_M \frac{\pi D^2}{4} (a_1\,\sigma_1^2 + a_2\,\sigma_2^2) \tag{20}$$

First and Second-Order Motions

The water particle velocity, u, for the first-order and second-order waves is $(u_I + u_2 + u_{II})$ and the water particle acceleration, $\dot{u}$, is $(\dot{u}_1 + \dot{u}_2 + \dot{u}_{II})$. By selecting the wave crest phase of the two deep water wave trains and the mean water level position (z = 0), the structural response due to drag force is

$$\zeta_{2c} = \frac{\varrho}{2M\omega_O^2} C_D\, D\left[a_1\,\sigma_1 + a_2\,\sigma_2 + |\vec{k_1} - \vec{k_2}|\,A - |\vec{k_1} + \vec{k_2}|\,B\right]^2 \tag{21}$$

If the mean water level phase of the two wave trains and mean water level position (z = 0) is chosen, the inertia force acting on the structure is

$$\zeta_{2m} = \frac{\varrho}{M\omega_O^2} C_M \frac{\pi D^2}{4}\left[a_1\,\sigma_1^2 + a_2\,\sigma_2^2 + (\sigma_1 - \sigma_2)\,|\vec{k_1} - \vec{k_2}|\,A - (\sigma_1 + \sigma_2)\,|\vec{k_1} + \vec{k_2}|\,B\right] \tag{22}$$

First, Second, and Third-Order Motions

If the third-order waves are involved in the wave force, u becomes $(u_1 + u_2 + u_{II} + u_{21})$ and $\dot{u}$ is $(\dot{u}_1 + \dot{u}_2 + \dot{u}_{II} + \dot{u}_{21})$. By selecting the wave crest phase of the two deepwater wave trains, it is found that the drag force of the third-order waves does not exist, but the inertia force does. In other words, the wave force at the wave crest phase contains the drag force of the first-order and second-order waves, and the inertia force of the third-order waves. The structural response due to this wave force at mean water level position (z = 0) is

$$\zeta_{3c} = \frac{\varrho}{2M\omega_O^2} C_D\, D\left[a_1\,\sigma_1 + a_2\,\sigma_2 + |\vec{k_1} - \vec{k_2}|\,A - |\vec{k_1} + \vec{k_2}|\,B\right]^2 - \frac{\varrho}{2M\omega_O^2} C_M \frac{\pi D^2}{4} P\,|2\vec{k_1} - \vec{k_2}|\left(t - \frac{2\lambda}{\omega_O}\right) \tag{23}$$

If the mean water level phase of the two wave trains is selected, it is found that only drag force of the third order waves remains. In other words, the drag force of the third-order waves will affect the structural response in addition to the inertia force of the first- and the second-order waves. Therefore, the structural response at mean water level position (z = 0) will be

$$\zeta_{3m} = \frac{\varrho}{2M\omega_O^2} C_D\, D\left[\frac{P}{2(2\sigma_1 - \sigma_2)}\,|2\vec{k_1} - \vec{k_2}|\right]^2 \left[\left(t - \frac{2\lambda}{\omega_O}\right)^2 - \frac{2}{\omega_O^2}(1 - 2\lambda^2)\right] + \frac{\varrho}{M\omega_O^2} C_M \frac{\pi D^2}{4}\left[a_1\,\sigma_1^2 + a_2\,\sigma_2^2 + (\sigma_1 - \sigma_2)\,|\vec{k_1} - \vec{k_2}|\,A - (\sigma_1 + \sigma_2)\,|\vec{k_1} + \vec{k_2}|\,B\right] \tag{24}$$

It can be seen that the third-order motions grow in time, Eqs. (23) and (24), but the second-order motions are bounded in time, Eqs. (21) and (22).

Nonlinear Wave Interaction Effects

The comparisons of structural response with and without nonlinear wave interactions are based on two different categories: (1) at the same wave phase and (2) of the same type of wave force.

The first category consists of several comparisons. If the wave crest phase is selected, the ratio of the structural response with and without the second-order waves can be defined as R_{2c}. Dividing Eq. (21) by Eq. (19), R_{2c} becomes

$$R_{2c} = \left(1 + \frac{|\vec{k_1} - \vec{k_2}|\,A - |\vec{k_1} + \vec{k_2}|\,B}{a_1\,\sigma_1 + a_2\,\sigma_2}\right)^2 \tag{25}$$

The ratio of the structural response with and without third-order waves is denoted as R_{3c} and is derived using Eqs. (23) and (19)

$$R_{3c} = \frac{1}{(a_1\sigma_1 + a_2\,\sigma_2)^2}\left\{\left[a_1\,\sigma_1 + a_2\,\sigma_2 + |\vec{k_1} - \vec{k_2}|\,A - |\vec{k_1} + \vec{k_2}|\,B\right]^2 - \left(\frac{C_M}{C_D}\right)\left(\frac{\pi D}{4}\right) P\,|2\vec{k_1} - \vec{k_2}|\left(t - \frac{2\lambda}{\omega_O}\right)\right\} \tag{26}$$

By choosing the mean water level phase, the ratio of the structural response with and without the second-order waves is denoted as R_{2m}. From Eqs. (20) and (22), it is

$$R_{2m} = 1 + \frac{(\sigma_1 - \sigma_2)\,|\vec{k_1} - \vec{k_2}|\,A - (\sigma_1 + \sigma_2)\,|\vec{k_1} + \vec{k_2}|\,B}{a_1\,\sigma_1^2 + a_2\,\sigma_2^2} \tag{27}$$

The ratio of the structural response with and without the third-order waves is R_{3m} which is derived from Eqs. (20) and (24) as

$$R_{3m} = 1 + \left(\frac{1}{a_1 \sigma_1^2 + a_2 \sigma_2^2}\right)\Bigg\{ (\sigma_1 - \sigma_2) \mid \vec{k_1} - \vec{k_2} \mid A - (\sigma_1 + \sigma_2) \mid \vec{k_1} + \vec{k_2} \mid B + \left(\frac{C_D}{C_M}\right)\left(\frac{2}{\pi D}\right) \left[\frac{P}{2(2\sigma_1 - \sigma_2)^2} \mid 2\vec{k_1} - \vec{k_2} \mid \right]^2 \left[\left(t - \frac{2\lambda}{\omega_o}\right)^2 - \frac{2}{\omega_o^2}(1 - 2\lambda^2)\right] \Bigg\} \tag{28}$$

Concerning the second category, if the drag force is focused, the ratio of the structural response with and without the second-order waves is represented by Eq. (25). The ratio of the structural response due to the third-order wave effects is denoted as Q_D, which can be derived using Eq. (19) and the first term on the right-hand side of Eq. (24)

$$Q_D = \left[\frac{P}{2(2\sigma_1 - \sigma_2)(a_1 \sigma_1 + a_2 \sigma_2)} \mid 2\vec{k_1} - \vec{k_2} \mid \right]^2 \left[\left(t - \frac{2\lambda}{\omega_o}\right)^2 - \frac{2}{\omega_o^2}(1 - 2\lambda^2)\right] \tag{29}$$

Similarly, the inertia force, Eq. (27) gives the ratio of the structural response due to the second-order wave effects. Q_I will be defined as the ratio of the structural response due to third-order effects. By using the second term of the right-hand side of Eq. (23) and Eq. (20), Q_I becomes

$$Q_I = \frac{\left(P \mid 2\vec{k_1} - \vec{k_2} \mid \right)\left(t - \frac{2\lambda}{\omega_o}\right)}{2(a_1 \sigma_1^2 + a_2 \sigma_2^2)} \tag{30}$$

where the " – " sign of the second term of Eq. (23) has been eliminated because only the magnitude is concerned.

4. RESULTS

To show the ratio of structural response with and without the presence of nonlinear wave interactions, some characteristic values of waves and structures have to be selected. The wave characteristic values chosen are $a_1 = 5$ ft., $a_2 = 3$ ft., $\sigma_1 = 0.628$ rad/sec (the dominant wave frequency in a typical hurricane), $\sigma_2 = 0.314 \sim 0.942$ rad/sec, $C_D = 1.0$ and $C_M = 1.4$. The values of structural characteristics are $D = 1$ ft., $\lambda = 0.05$ and $\omega_O = 1.3$ rad/sec (the natural frequency of the Cognac platform installed in 1025 ft. of water, see Sterling et al. [7]). The ratios, R_{2c} and R_{2m}, are found to be almost equal to unity. Hence the second-order waves only produce a small modification to the structural motion. However, the structural motion due to the third-order effect is found to be significant.

Figure 3 shows the variation of R_{3c} with respect to the angle between the two wave trains, θ, for time scales "T" 50 and 100 times the wave period of the first wave train (*i.e.*, T_1). It can be seen that for $\epsilon < 0$ (*i.e.*, $\sigma_2 < \sigma_1$), the ratio of the structural response with and without the third-order waves at wave crest phase is smaller than for $\epsilon > 0$ (*i.e.*, $\sigma_2 > \sigma_1$). The magnitude of R_{3c} is less than unity. In other words, the structural response with the effect of the third-order waves at wave crest phase decreases with increasing the time scales. It is because the direction of the horizontal acceleration of the third-order waves is opposite to that of the horizontal velocities of the first and second-order waves. A minimum value of R_{3c} occurs at $\theta = 17°$ when σ_2 is smaller than σ_1. The minimum values are 0.93 for time scales 50 times the wave period of the first wave train and 0.85 for time scales 100 times the wave period.

The variation of R_{3m} versus the angle between the two wave trains, θ, is shown in Figure 4. By using the same time scales as R_{3c}, the ratio, R_{3m}, of the structural response with and without the third-order waves at mean water level phase, for $\epsilon < 0$ is greater than that for $\epsilon > 0$. The structural response with the effect of the third-order waves at mean water level phase increases when the time scales increases. As σ_2 is smaller than σ_1, maximum values of R_{3m}, occuring at $\theta = 6°$, are 1.19 and 1.75 for the cases of $T = 50$ and 100 wave period of the first wave train.

The ratio, Q_D, is shown plotted against θ in Figure 5. It is greater when $\epsilon < 0$, and has a maximum near $\theta = 6°$. As the time scales are 50 and 100 of T_1, the maximum values of Q_D are 0.052 and 0.21. Q_I against θ is shown in Figure 6. Like Q_D, it is greater over the range of $\epsilon < 0$ and the maximum occurs at $\theta = 17°$. The maximum values are 0.26 for $T = 50\ T_1$ and 0.53 for $T = 100\ T_1$.

The results show that the accretion of the structural response at mean water level phase is larger than the reduction of the structural response at wave crest phase. It is worthwhile to discuss whether the structural response due to the third-order waves at mean water level phase would be larger than that at wave crest phase. First, the results show that the structural response due to the drag force at mean water level (third-order waves) compared to that at wave crest phase (first-order waves), *i.e.*, Figure 5, is much smaller than the structural response due to the inertia force at wave crest phase (third-order waves) compared to that at mean water level (first-order waves), *i.e.*, Figure 6. Therefore, the real variation of the structural response at mean water level phase would not be as significant as the R_{3m} shows. Secondly, the magnitude of the structural response at wave crest phase is greater due to the additional submergence of structures at the wave crest phase (a contribution not included in the present results). Therefore, the structural response at the wave crest phase is still larger than that at the mean water level phase.

5. CONCLUSIONS

The effect of the nonlinear wave interactions of deepwater waves play an important role on the dynamic response of fixed offshore structures. The results show that the second-order waves (cross interactions) only make a small modification to the structural response. The third-order waves (resonant interactions), however, produce a significant effect because they grow in time. The structural response due to the effect of the third-order waves at the wave crest phase decreases with increasing time of resonant interactions. At mean water level phase, the structural response with the third-order wave effects increases as the time scale increases. For both wave phases, the variations of the structural response as $\sigma_2 < \sigma_1$ is larger than those as $\sigma_2 > \sigma_1$. Also, the variations vanish when the two wave trains are parallel or anti-parallel.

ACKNOWLEDGMENTS

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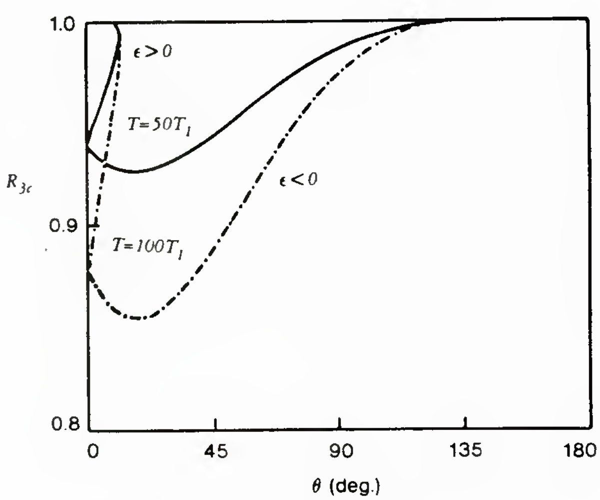

Figure 3. Variations of structural response due to resonant interactions at wave crest phase with θ

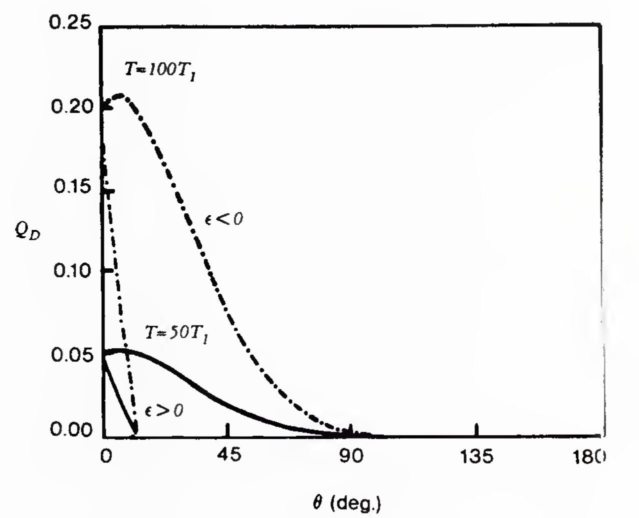

Figure 5. Variations of structural response due to drag force of resonant interactions with θ.

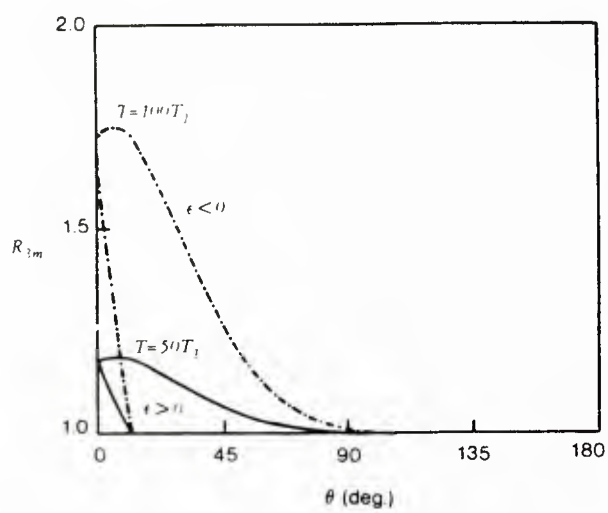

Figure 4. Variations of structural response due to resonant interactions at mean water level phase with θ

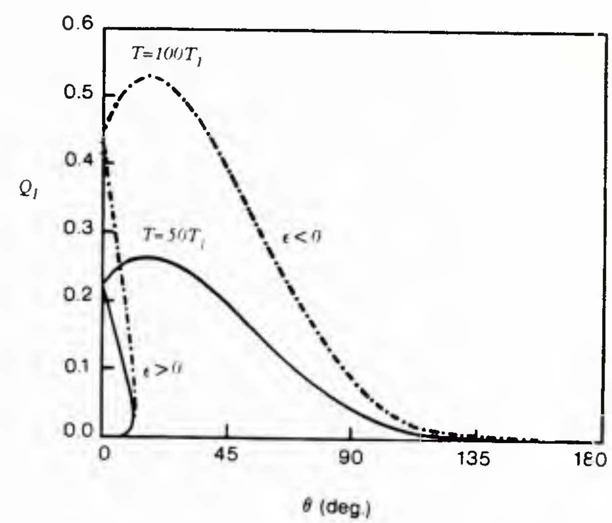

Figure 6. Variations of structural response due to inertia force of resonant interactions with θ.

Appendix B: Random wave force transformation on sloping beach

Random Wave Force Transformation on Sloping Beach

Benjamin Yun-Hai Chen[1], A.M. ASCE, and Cheng Y. Yang[2], M. ASCE

Abstract

A linear random wave force transformation model on a sloping beach is developed. Based on the random process approach, this model develops a new analytical solution concerning random wave force transformation on a sloping beach. A new derivation of linearizing the drag force term also is obtained. By comparing the results, it is found that this new model, and the approximate model, shows a general agreement in spatial variation, with a difference varying from 5-8%.

Introduction

Most of the coastal structures, may sustain severe damage due to hurricanes or storms sweeping through a coastal region. To design these coastal structures for satisfactory operation during strong wave conditions, an accurate prediction of the storm-generated wave forces on structures is necessary.

Borgman (1) developed an approximate random wave force model that assumes the local slope of the beach to be horizontal. Yang and Chen (5) developed an analytical solution to predict the change of energy as random waves propagation from deep water towards the sloping beach. This paper, based on the random process approach, presents the development of a linear random wave force transformation model on a sloping beach from a given storm-generated wave field in deep water.

Wave Transformation for a Locally Flat Bottom

Consider a linear single wave propagating normally to the shoreline. The velocity potential $\Phi(x,y,t)$ is given by

$$\Phi(x,y,t) = -a\,\frac{g}{\sigma}\,\frac{\cosh k\,(h+y)}{\cos h\,k\,h}\,\sin\,(kx-\sigma t+\theta) \qquad (1)$$

in which k and σ represent the wave number and angular frequency; θ, a, and h are the phase, amplitude, and water depth (g is the acceleration of gravity, x is towards the beach and y is upward). The associated water surface displacement $\eta(x,t)$ is

$$\eta(x,t) = a\,\cos\,(kx-\sigma t+\theta) \qquad (2)$$

Assume that the phase θ is random and distributed uniformly in $(0,2\pi)$. The ensembel mean of the random process, $\eta(x,t)$, is zero and the variance

$$\mathrm{var}\,[\eta(x,t)] = 1/2\,a^2 = S_\eta\,(\sigma)\,\Delta\sigma \sim E \qquad (3)$$

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in which E and $S\eta\ (\sigma)$ represent the wave energy and the wave frequency spectral density, respectively.

Therefore, the wave energy transformation can be represented by

$$S\eta\ (\sigma) = A_1(h)\ S\eta_o\ (\sigma) \tag{4}$$

in which

$$A_1\ (h) = \frac{E}{E_o} = \frac{C_{Go}}{C_G} \tag{5}$$

represents the energy transfer function. C_G is the group velocity and "o" refers to deep water. The techniques of random process are used hereafter, but the details are not shown.

The horizontal velocity and acceleration transformations are

$$Su\ (\sigma) = B_1(h,y)\ S\eta_o\ (\sigma) \tag{6}$$

$$Sa_x\ (\sigma) = C_1(h,y)\ S\eta_o\ (\sigma) \tag{7}$$

in which $B_1\ (h,\ y) = \dfrac{\sigma^2 \cosh^2 k\ (h+y)}{\sinh^2 kh}\ A_1(h)$ (8)

and

$$C_1\ (h,\ y) = \frac{\sigma^4 \cosh^2 k\ (h+y)}{\sinh^2 kh}\ A_1(h) \tag{9}$$

represent the horizontal velocity and acceleration transfer functions.

The wave force per unit length on a vertical cylinder can be calculated from the Morison's formula

$$F = J\ u|u| + Na_x \tag{10}$$

in which $J = C_D\ \frac{\rho}{2}\ D$ and $N = C_M\ \rho\ \frac{\pi D^2}{4}$. C_D and C_M represent the drag and the inertial coefficients. D and ρ are the diameter of the vertical cylinder and the fluid density.

It is intractable to calculate the wave force due to the nonlinear term in the drag force. Borgman (1, 2) derived two approaches, series representation and optimal representation, to linearize the drag force term for a Gaussian random model. In this paper, based on the optimal representation approach, the linearization of the drag force term for a single component wave is developed. Assume the phase θ to be random with uniform distribution in $(0,\ 2\pi)$. The linear approximation of $u|u|$ can be derived as

$$u|u| \approx \left(\frac{8\sqrt{2}}{3\pi}\ u_{rms}\right) u \tag{11}$$

in which u_{rms} is the standard deviation of velocity u.

Inserting Eq. (11) to Eq. (10) and seeking the ensemble mean and variance of F (x, y, t), the wave force transformation yeilds

$$S_F\ (\sigma) = G_1\ (h,\ y)\ S\eta_o\ (\sigma) \tag{12}$$

where $G_1\ (h,\ y) = \dfrac{128}{9\pi^2}\ J^2\ B_1{}^2(h,\ y)\ \dfrac{a_o{}^2}{2} + N^2\ C_1(h,y)$ (13)

represents the wave force transfer function. a_o is the wave amplitude in deep water.

For a multiple of n component waves, assume the random phase θn to be independently and uniformly distributed in (o, 2π). Under the assumption of independent propagation for all component waves, the transfer functions for each component wave are governed by the same solutions as the single wave. More details can be seen in Chen (3).

Wave Transformation for a Nonshallow Sloping Beach

Stoker (4) solved the problem of deterministic progressive waves over a uniformly sloping bottom with the sloping angle $\pi/2n$ (n is an integer). For a nonshallow sloping beach of 45°, the velocity potential $\Phi(x, y, t)$ is given by

$$\Phi(x,y,t) = \frac{a_o g}{\sigma} \left[\frac{\sqrt{2}}{\pi} \phi_1 (x,y) \cos (\sigma t + \theta) + \frac{1}{\pi} \phi_2 (x,y) \sin (\sigma t + \theta)\right] \tag{14}$$

in which $\phi_1 (x,y) = \frac{\pi}{\sqrt{2}} \left[e^{-k_o x} \cos (\frac{\pi}{4} - k_o y) + e^{k_o y} \cos (\frac{\pi}{4} + k_o x)\right]$ (15)

is a regular standing wave solution and

$$\phi_2 (x,y) = \frac{e^{k_o y}}{\sqrt{2}} [C_i (k_o x) [\sin (k_o x) - \cos (k_o x)] - [\frac{\pi}{2} + S_i (k_o x)] \tag{16}$$

$$[\cos (k_o x) + \sin (k_o x)] - e^{-k_o x} Ei (k_o x)]$$

is a singular standing wave solution. $S_i (k_o x)$, $C_i (k_o x)$ and $E_i (k_o x)$ represent sine, cosine, and exponential integral functions, respectively.

The wave energy transformation can be expressed by

$$S_\eta (\sigma) = A (x) S_{\eta_o} (\sigma) \tag{17}$$

in which $A (x) = \frac{1}{\pi^2} [2\phi_1^2 (x,o) + \phi_2^2 (x,o)]$ (18)

is the energy transfer function. Eq. (18) has been shown in Yang and Chen (5).

The horizontal velocity and acceleration transformations become

$$Su (\sigma) = B (x, y) S_{\eta_o} (\sigma) \tag{19}$$

$$S_{a_x} (\sigma) = C (x, y) S_{\eta_o} (\sigma) \tag{20}$$

in which $B (x, y) = \frac{g^2}{\pi^2 \sigma^2} \left[2 \left(\frac{\partial \phi_1 (x, y)}{\partial x}\right)^2 + \left(\frac{\partial \phi_2 (x, y)}{\partial x}\right)^2 \right]$ (21)

$$C (x, y) = \frac{g^2}{\pi^2} \left[2 \left(\frac{\partial \phi_1 (x, y)}{\partial x}\right)^2 + \left(\frac{\partial \phi_2 (x, y)}{\partial x}\right)^2 \right] \tag{22}$$

represent the horizontal velocity and acceleration transfer functions.

Using the linear approximation of the drag force term, the wave force transformation yields

$$S_F (\sigma) = G (x, y) S_{\eta_o} (\sigma) \tag{23}$$

in which $G (x, y) = \frac{128}{9\pi^2} J^2 B^2(x, y) \frac{a_o^2}{2} + N^2 C (x,y)$ (24)

stands for the wave force transfer function. The derivations of n random wave components are given in Chen (3).

Results

To show the comparisons of the transfer function between locally flat bottom and nonshallow sloping beach, the wave transfer function has been normalized by $\sigma^4 \left(\frac{128}{9\pi^2} J^2 \frac{a_o^2}{2} + N^2\right)$. The values of C_D, C_M, and D have been chosen as 1.05, 1.4 and 1 ft (0.33 m). Figure 1 illustrates the comparisons of the normalized wave force transfer function between two cases. The comparisons show a general agreement in their spatial variation. The maximum difference varies from 5% to 8% at $k_o x = 2.3$ as a_o increases from 1 ft (0.33 m) to 10 ft (3.33 m).

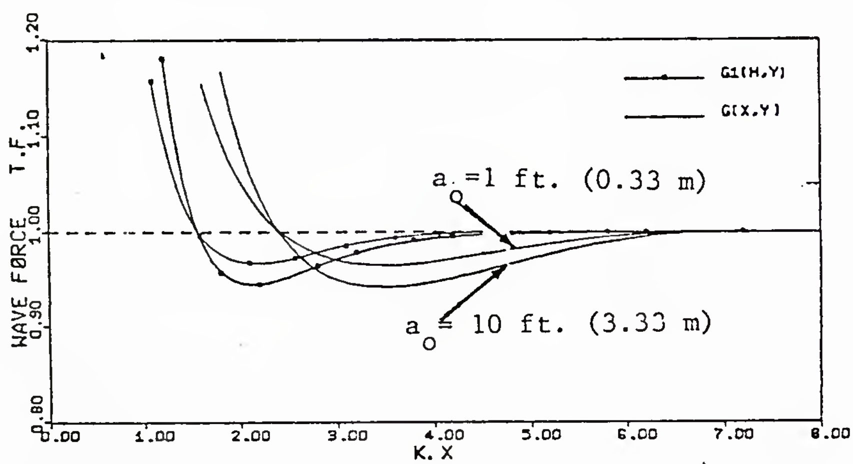

Figure 1 The Comparisons of Wave Force Transfer Function at y = 0

Conclusions

A linear random wave force model on sloping beach is developed. This model improves the current approximate model because the assumption of a locally flat bottom is removed. Though the results only show in 45° sloping beach, they can extend to $\pi/2n$ sloping beach (with n an integer). For engineering application, this model is used to predict wave force spectra at specified locations in the coastal region from given deepwater wave energy spectra.

Acknowledgements

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Appendix C: Wave group effects on offshore structures

WAVE GROUP EFFECTS ON OFFSHORE STRUCTURES

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ABSTRACT

The effect of wave groups and an associated forced second-order wave system on a fixed offshore structure is examined analytically. Based on the modified Morison's equation approach, the dynamic response of a single degree of freedom, fixed offshore structure is investigated using a primary wave system with and without the second-order wave system. By comparing the results, it is found that the second-order wave system can significantly affect structural response. This is particularly true of structures in shallow water.

1. INTRODUCTION

Most methods for calculating the dynamic response of off-shore structures use a spectrum of waves as the forcing function. Examples include the works of Chakrabarti and Cotter [1], Chung et al. [2], Kirk and Etok [5], Pinkster [8], and Vandiver [9]. If the spectrum is reasonably narrow, then fairly well-defined "wave groups" can result. For design purposes, it is important to consider the effect of the largest waves in the groups since they can cause structural damage. An approach based on individual waves of the same size and dispersed throughout a wave train is a less severe case.

Longuet-Higgins [6] and Longuet-Higgins and Stewart [7] have shown that wave groups drive a coupled, second-order wave system. The effect of this wave theory on the dynamic response of a fixed offshore structure is analytically investigated in this paper. Generally the determination of wave loads on offshore structures is based on two major techniques. For large structures, the scattering of the incident waves is considered and a diffraction theory is employed. The wave loads on small members of offshore structures are usually determined by applying Morison's equation, which includes drag and inertia force components. For simplicity, a single degree of freedom structure with negligible wave scattering is considered. Also, the wave loads are determined by applying the modified Morison equation.

2. WAVE GROUP THEORY

The wave group model of Longuet-Higgins and Stewart is illustrated in Figure 1. Its basic components include a primary wave system and a second-order wave system that is always out of phase with the group amplitude. This phenomenon is illustrated in Figure 1 where the second-order wave trough is shown at the wave group maxima and the second-order wave crest at the wave group minima.

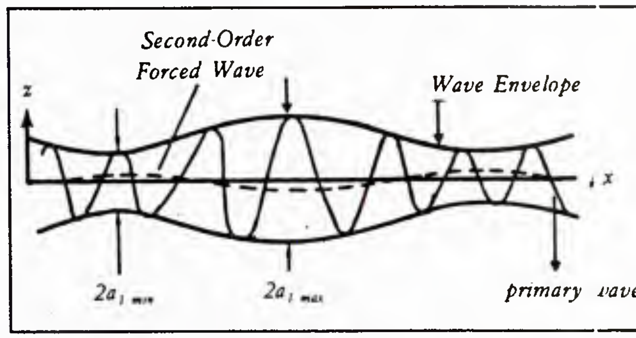

Figure 1. Wave due to second-order forced wave system in a wave group.

In their papers, Longuet-Higgins and Stewart investigate two cases: (1) the primary waves are relatively short compared to the depth and (2) the primary waves are not necessarily short, but the "group length" is long compared to the depth. In this paper, the latter case's effect on structural response will be investigated since it is the most significant from wave force considerations.

The mathematical models of the wave group components are given below and are based on the coordinate system shown in Figure 1.

Primary Wave System

The velocity potential, ϕ_1, water surface, σ_1, horizontal velocity, u_1, and acceleration, $\dot{u}_1$, for the primary wave system are

$$\phi_1(x,z,t) = \frac{a_1\sigma}{k}\frac{\cosh k(h+z)}{\sinh kh}\sin(kx-\sigma t) \quad (1)$$

$$\eta_1(x,t) = a_1\cos(kx-\sigma t) \quad (2)$$

$$u_1(x,z,t) = \frac{\partial\phi_1}{\partial x} = a_1\sigma\frac{\cosh k(h+z)}{\sinh kh}\cos(kx-\sigma t) \quad (3)$$

$$\dot{u}_1(x,z,t) = \frac{\partial u_1}{\partial t} = a_1\sigma^2\frac{\cosh k(h+z)}{\sinh kh}\sin(kx-\sigma t) \quad (4)$$

where k is the wave number, σ is the angular frequency, h is the water depth, and a_1 is the "local" wave amplitude dependent on x.

Second-Order Wave System

Velocity potential, ϕ_2, for the second-order wave system is

$$\phi_2(x,z,t) = -a_2 C_g \frac{\cosh \Delta k(h + z)}{\sinh \Delta kh} \sin \Delta k(x - C_g t) \tag{5}$$

where C_g is the group velocity, Δk is the "group" wave number and a_2 is given by

$$a_2 = \frac{(a_{1_{max}}^2 - a_{1_{min}}^2)}{4h\left(1 - \frac{C_g^2}{gh}\right)} \left(1/2 + \frac{2kh}{\sinh 2kh} \right) \tag{6}$$

Here, $a_{1\,max}$ is the maximum displacement of the wave envelope and $a_{1\,min}$ the minimum displacement of the wave envelope. Both parameters are shown in Figure 1.

The water surface, η_2, for the second-order wave system is given as

$$\eta_2(x,t) = -a_2 \cos \Delta k(x - C_g t) \tag{7}$$

When $\cos \Delta k\ (x - C_g t) = 1$, η_1 is a maximum and η_2 a minimum.

The horizontal velocity, u_2, and acceleration, $\dot{u}_2$, of the second-order wave system are given as

$$u_2(x,z,t) = \frac{\partial \phi_2}{\partial x} = -a_2 C_g \Delta k \frac{\cosh \Delta k(h + z)}{\sinh \Delta kh} \cos \Delta k(x - C_g t) \tag{8}$$

and

$$\dot{u}_2(x,z,t) = \frac{\partial u_2}{\partial t} = -a_2 C_g^2 \Delta k^2 \frac{\cosh \Delta k(h + z)}{\sinh \Delta kh} \sin \Delta k(x - C_g t) \tag{9}$$

which are both seen to be negative when the wave envelope is a maximum.

Since the dynamic response of an offshore structure would be affected by the waves forming the extremes of the wave envelopes, the effect of these second-order waves are considered in this paper.

3. EFFECT OF SECOND-ORDER WAVE SYSTEM ON STRUCTURES

For a generalized, single degree of freedom system of a fixed offshore structure, the equation of motion is

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = F_W(t) \tag{10}$$

where M is the mass of the structure, C the internal structural damping, K the structural stiffness, X, $\dot{X}$ and $\ddot{X}$ are, respectively, the displacement, velocity, and acceleration of the structure and $F_W(t)$ is the external force acting on the structure. All the above quantities are based on a unit length of the structures. $F_W(t)$ is represented by the modified Morison's equation:

$$F_W(t) = \frac{\varrho}{2} C_D D \mid u(t) - \dot{X}(t) \mid [u(t) - \dot{X}(t)] + \varrho \frac{\pi D^2}{4} \dot{u}(t) + \varrho (C_M - 1) \frac{\pi D^2}{4} [\dot{u}(t) - \ddot{X}(t)] \tag{11}$$

where C_D is the drag coefficient, C_M the inertia coefficient, $(C_M - 1)$ the added mass coefficient, ϱ the mass density of water, D the pile diameter, u the local water particle velocity, and $\dot{u}$ the acceleration.

For most wave-loading conditions, the water particle velocity is much larger than that of the structure. Hence, the following approximation of Dao and Penzin [3] can be made.

$$\mid u(t) - \dot{X}(t) \mid [u(t) - \dot{X}(t)] \approx u(t) \mid u(t) \mid - 2 \mid u(t) \mid \dot{X}(t) \tag{12}$$

Inserting Eqs. (11) and (12) into Eq. (10), then, gives the equation of motion as

$$\left[M + (C_M - 1)\, \varrho \frac{\pi D^2}{4} \right] \ddot{X}(t) + C\dot{X}(t) + \varrho C_D D \mid u(t) \mid \dot{X}(t) + KX(t) = \frac{\varrho}{2} C_D D\, u(t) \mid u(t) \mid + \varrho C_M \frac{\pi D^2}{4} \dot{u}(t) \tag{13}$$

Drag Force Effects

To find the total displacement of the structure due to drag force, *i.e.*, the first term on the right side of Eq. (13), the crest phase of the primary wave system must be selected. By integrating Eq. (13) from the ocean bottom (z = −h) up to the mean free surface, (z = 0), the total displacement of the structure due to drag force and the primary wave system (X^{*}_{DWO}) is

$$X^{*}_{DWO} = \frac{\varrho}{2K^{*}} C_D D a_{1_{max}}^2 \frac{\sigma^2}{\sinh^2 kh} \left(\frac{\sinh 2kh + 2kh}{4k} \right) \tag{14}$$

where K^{*} is the total structure stiffness.

The total displacement of the structure due to drag force with the primary and second-order wave systems (X^{*}_{DW}) is

$$X^{*}_{DW} = \frac{\varrho}{2K^{*}} C_D\ D \left\{ a_{1_{max}}^2 \frac{\sigma^2}{\sinh^2 kh} \frac{(\sinh 2kh + 2kh)}{4k} - 2 \frac{a_{1_{max}} a_2 \sigma C_g \Delta k}{\sinh kh \sinh \Delta kh} \left(\frac{k \sinh kh \cosh \Delta kh - \Delta k \cosh kh \sinh \Delta kh}{k^2 - \Delta k^2} \right) + a_2^2 C_g\ \Delta k \frac{(\sinh 2\Delta kh + 2\Delta kh)}{4 (\sinh \Delta kh)^2} \right\} \tag{15}$$

where a_1 in Eqs. (14) and (15) has been taken as the maximum value of the envelope.

Inertia Force Effects

The total displacement of the structure due to inertia force, *i.e.*, the second term on the right side of Eq. (13), occurs at the mean water level. By integrating Eq. (13) from the ocean bottom ($z = -h$) up to the mean free surface ($z = 0$), the total displacement of the structure due to inertia force without (X^{*}_{IWO}) and with (X^{*}_{IW}) the effect of second-order wave system is computed as

$$X^{*}_{IWO} = \frac{\varrho}{K^{*}} C_M \frac{\pi D^2}{4} a_1 \frac{\sigma^2}{k} \tag{16}$$

and

$$X^{*}_{IW} = \frac{\varrho}{K^{*}} C_M \frac{\pi D^2}{4} \left[a_1 \frac{\sigma^2}{k} - a_2 C_g^2 \Delta k \right] . \tag{17}$$

Second-Order Wave System Effects

The total displacement of the structure due to the second-order wave system increases with decreasing water depth because the primary wave system transfers energy into the second-order wave system. In discussing this effect a_{WO} and a_W will denote, respectively, the primary wave system amplitudes without and with the energy transfer to the second-order wave system taken into account, *i.e.*, $a_{WO} > a_W$.

The ratio, R_{DX}, of the total displacement of the structure due to drag force with and without the presence of the second-order wave system is obtained by dividing Eq. (15) by Eq. (14):

$$R_{DX} = \frac{X^{*}_{DW}}{X^{*}_{DWO}} = \left(\frac{a_W}{a_{WO}}\right)^2 (1 - P_1 + P_2) \tag{18}$$

where

$$P_1 = \frac{a_W}{2h} \left\{ \frac{\frac{1}{2} + \frac{2kh}{\sinh 2kh}}{1 - \frac{C_g^2}{gh}} \right\} \frac{\tanh kh}{kh} \tag{19}$$

and

$$P_2 = \frac{1}{32} \left(\frac{a_W}{h}\right)^2 \left\{ \frac{\frac{1}{2} + \frac{2kh}{\sinh 2kh}}{1 - \frac{C_g^2}{gh}} \right\}^2 \left(1 + \frac{2kh}{\sinh 2kh}\right) \frac{\tanh kh}{kh} \tag{20}$$

In deriving Eq. (18) it is assumed that $k >> \Delta k$ and that $\Delta kh << 1$.

Similarly, the ratio of the total displacement of the structure due to inertia force with and without the presence of the second-order wave system, R_{IX}, is

$$R_{IX} = \frac{X^{*}_{IW}}{X^{*}_{IWO}} = \left(\frac{a_W}{a_{WO}}\right) (1 - Q) , \tag{21}$$

where

$$Q = \frac{a_W}{16\, h} \left\{ \frac{\frac{1}{2} + \frac{2kh}{\sinh 2kh}}{1 - \frac{C_g^2}{gh}} \right\} \left(1 + \frac{2kh}{\sinh 2kh}\right)^2 \frac{\Delta k}{k} \tag{22}$$

To evaluate the importance of the total displacement of the structure due to the second-order wave system (Eqs. (18) and (21)), it is necessary to establish the ratio, a_W/a_{WO}, *i.e.*, the effect of energy transfer from the primary to the second-order wave system. Dean [4] derived the ratios, a_W/a_{WO} and a_2/a_{WO}, by using the energy flux conservation between the primary wave system with and without the second-order wave system. They are given as:

$$\zeta = \frac{a_W}{a_{WO}} = \sqrt{\frac{-1 + \sqrt{1 + 8F^2 \left(\frac{a_{WO}}{h}\right)^2}}{4F^2 \left(\frac{a_{WO}}{h}\right)}} . \tag{23}$$

where

$$F\left(\frac{h}{L_o}\right) = \frac{1}{4\left(1 - \frac{C_g^2}{gh}\right)} \left(\frac{1}{2} + \frac{2kh}{\sinh 2kh}\right) \tag{24}$$

and

$$\frac{a_2}{a_{WO}} = \zeta^2 \frac{a_{WO}}{h} F\left(\frac{h}{L_o}\right) \tag{25}$$

4. RESULTS

Figure 2 shows the variations of a_W/a_{WO} and a_2/a_{WO} for a_{WO} = 0.25 and 1.0 of the breaking value, a_{WOB}. It is interesting that the second-order wave amplitude exceeds the primary wave amplitude in shallow water. In reality, the theory of the second-order wave system is valid for only relatively small second-order wave amplitudes, especially in shallow water. Therefore, a high-order wave theory is necessary if good accuracy is required. For discussion purposes, the results are considered valid if the ratio a_2/a_{WO} is on the order of 0.10 to 0.20.

The ratio, R_{DX}, for a_{WO} = 0.25 and 1.0 of the breaking value is shown in Figure 3. It is seen that this ratio varies significantly in shallow water. If the ratio a_2/a_{WO} = 0.1 is taken, the corresponding R_{DX} values are 0.79 and 0.88 for relative breaking wave amplitudes of 0.25 and 1.0, respectively. A value of a_2/a_{WO} = 0.2 yields values of R_{DX} = 0.61 for a_{WO} = 0.25 A_{WOB} and 0.55 for $A_{WO} = A_{WOB}$.

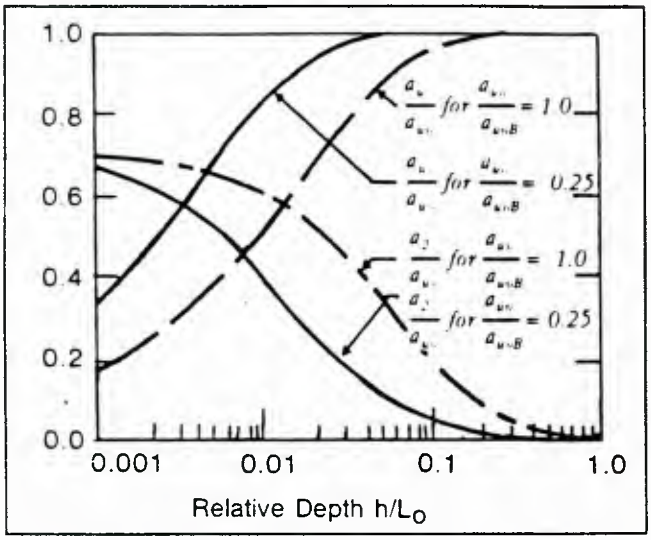

Figure 2. Variation of a_W/a_{WO} and a_2/a_{WO} with h/L_O for a_{WO} A_{WOB} = 0.25 and 1.0.

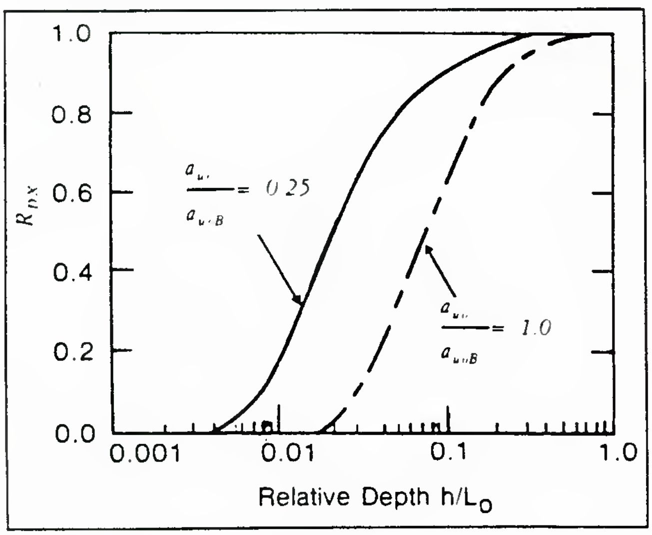

Figure 3. Ratio of total displacement of structures due to drag force for the wave crest position.

Figure 4 gives the ratio R_{IX} versus relative depth for $\Delta k/k$ = 0.1 (*e.g.*, the wave length of the primary wave system and the wave group are 100 m and 1 km) and a_{WO} = 0.25 a_{WOB} and a_{WOB}. It is seen that this ratio is small in the shallow-water range and approaches unity in deep water. The reduction of R_{IX} is from 2 to 6% as the second-order amplitude is from 10 to 20% of the primary wave system.

5. CONCLUSIONS

The effect of second-order wave system can significantly affect the dynamic response of fixed offshore structures. This is particularly true in shallow water. Structural response due to drag force and inertia force from the combined effects of the primary and the second-order wave systems are found to be reduced at the wave crest position and the mean water level. The reduction of structural response

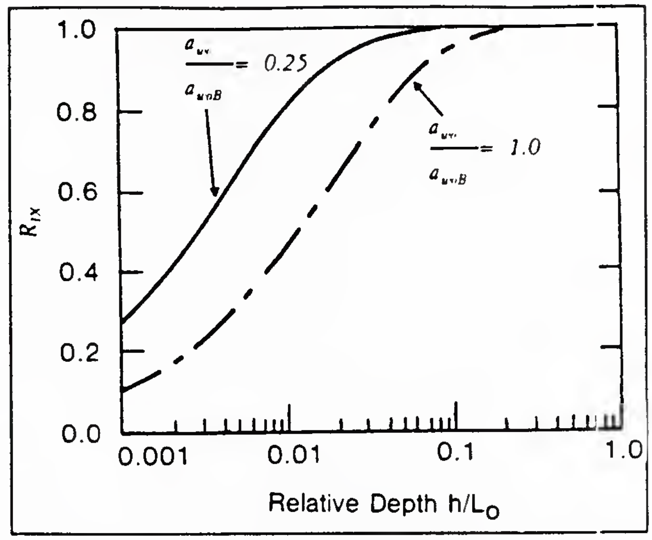

Figure 4. Ratio of total displacement of structures due to inertia force for the mean water level position and $\Delta k/k$ = 0.1.

due to inertia force is much smaller than that due to drag force because the drag force includes a square term. The structural response due to drag force at the trough position has not been shown in the results. Clearly, the results would be an increase of structural response at the trough phase position. However, it is because the magnitude of the structural response at crest position is greater than that at trough position due to the additional submergence of piling at the crest position. Therefore, only the crest position was considered.

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